Name:____ Date:________________________

Algebra 2 CC: Zero and Negative Exponents

In general, for $x \neq 0$ and n a positive integer:

$$\frac{x^n}{x^n} = x^{n-n} = x^0$$

In order for the rule for the division of powers with like bases to be consistent with ordinary division, we must be able to show that for all $x \neq 0$, $x^0 = 1$.

Multiplication:	$x^n \cdot x^0 = x^{n+0} = x^n$	$x^n \cdot x^0 = x^n \cdot 1 = x^n$
Division:	$x^n \div x^0 = x^{n-0} = x^n$	$x^n \div x^0 = x^n \div 1 = x^n$
Raising to a Power:	$(x^0)^n = x^{0 \cdot n} = x^0 = 1$	$(x^0)^n = 1^n = 1$
Power of a Product:	$(xy)^0 = x^0 \cdot y^0 = 1 \cdot 1 = 1$	$(xy)^0 = 1$
Power of a Quotient:	$\left(\frac{x}{y}\right)^0 = \frac{x^0}{y^0} = \frac{1}{1} = 1$	$\left(\frac{x}{y}\right)^0 = 1$

Therefore, it is reasonable to make the following definition:

DEFINITION

If $x \neq 0$, $x^0 = 1$.

Multiplication: $2^5 \cdot 2^{-3} = 2^{5+(-3)} = 2^2$

$2^5 \cdot 2^{-3} = 2^5 \cdot \frac{1}{2^3} = 2^2$

Division: $6^2 \div 6^{-5} = 6^{2-(-5)} = 6^7$

$6^2 \div 6^{-5} = 6^2 \div \frac{1}{6^5} = 6^2 \cdot 6^5 = 6^7$

Raising to a Power: $(3^{-4})^{-2} = 3^{-4(-2)} = 3^8$

$(3^{-4})^{-2} = \left(\frac{1}{3^4}\right)^{-2} = \frac{1}{3^{-8}} = \frac{1}{\frac{1}{3^8}} = \frac{3^8}{1} = 3^8$

Power of a Product: $(x^2y^{-3})^4 = x^8y^{-12} = x^8 \cdot \frac{1}{y^{12}} = \frac{x^8}{y^{12}}$

$(x^2y^{-3})^4 = (x^2 \cdot \frac{1}{y^3})^4 = (x^2)^4\left(\frac{1}{y^3}\right)^4 = \frac{x^8}{y^{12}}$

Power of a Quotient: $\left(\frac{x^3}{y^5}\right)^{-2} = \frac{x^{-6}}{y^{-10}} = \frac{\frac{1}{x^6}}{\frac{1}{y^{10}}} = \frac{y^{10}}{x^6}$

$\left(\frac{x^3}{y^5}\right)^{-2} = \frac{1}{\left(\frac{x^3}{y^5}\right)^2} = \frac{1}{\frac{x^6}{y^{10}}} = \frac{y^{10}}{x^6}$

Therefore, it is reasonable to make the following definition:

DEFINITION

If $x \neq 0$, $x^{-n} = \frac{1}{x^n}$.

Developing Skills

In 3–10, write each expression as a rational number without an exponent.

3. 5^{-1} **4.** 4^{-2} **5.** 6^{-2} **6.** $\left(\frac{1}{2}\right)^{-1}$

7. $\left(\frac{1}{5}\right)^{-3}$ **8.** $\left(\frac{2}{3}\right)^{-1}$ **9.** $\frac{3^0}{4^{-2}}$ **10.** $\frac{(2 \cdot 5)^{-4}}{5^{-2}}$

In 11–22, find the value of each expression when $x \neq 0$.

11. 7^0 **12.** $(-5)^0$ **13.** x^0 **14.** -4^0

15. $(4x)^0$ **16.** $4x^0$ **17.** $-2x^0$ **18.** $(-2x)^0$

19. $\left(\frac{3}{4}\right)^0$ **20.** $\frac{3^0}{4}$ **21.** $\frac{3^0}{4^0}$ **22.** $\frac{3x^0}{(4x)^0}$

In 23–34, evaluate each function for the given value. Be sure to show your work.

23. $f(x) = x^{-3} \cdot x^4; f(1)$ **24.** $f(x) = x + x^{-5}; f(3)$

25. $f(x) = (2x)^{-6} \div x^3; f(-3)$ **26.** $f(x) = (x^{-7})^4; f(-6)$

27. $f(x) = \left(\frac{1}{x} + \frac{3}{2}\right)^{-2}; f(2)$ **28.** $f(x) = 10^x + 10^{-2x}; f(3)$

29. $f(x) = x^{-7} \div x^8; f\left(\frac{3}{4}\right)$ **30.** $f(x) = (3x^{-3} - 2x^{-3})^2; f(-2)$

31. $f(x) = x^8\left(x^{-2} + \frac{1}{x^3}\right); f\left(\frac{1}{2}\right)$ **32.** $f(x) = \left(\frac{x^{-1}}{(2x)^{-2}}\right)^{-1}; f(8)$

33. $f(x) = \frac{1}{1 + \frac{2}{x^{-1}}}; f(-5)$ **34.** $f(x) = 4\left(\frac{1}{2}\right)^{-x} + 3\left(\frac{1}{2}\right)^{-x}; f(3)$

In 35–63, write each expression with only positive exponents and express the answer in simplest form. The variables are not equal to zero.

35. x^{-4} **36.** a^{-6} **37.** y^{-5} **38.** $2x^{-2}$

39. $7a^{-4}$ **40.** $-5y^{-8}$ **41.** $(2x)^{-2}$ **42.** $(3a)^{-4}$

43. $(4y)^{-3}$ **44.** $-(2x)^{-2}$ **45.** $-(3a)^{-4}$ **46.** $(-2x)^{-2}$

47. $\frac{1}{x^{-3}}$ **48.** $\frac{1}{y^{-7}}$ **49.** $\frac{3}{a^{-5}}$ **50.** $\frac{6}{a^{-4}}$

51. $\frac{9x^2}{a^{-3}}$ **52.** $\frac{(-x)^{-5}}{x^{-3}}$ **53.** $\frac{(2a)^{-1}}{2(a)^{-2}}$ **54.** $\frac{4y^{-3}}{2y^{-1}}$